MATH 1650: ABSOLUTE VALUE REVIEW

EXAMPLE: Solve each of the following equations.

1. Solve $|3x-1|=6$:

   The equation $|3x-1|=6$ is in the form $|X|=c$, so we know that either $3x-1=6$ or $3x-1=-6$.

   Solving $3x-1=6$ gives $x=\frac{7}{3}$ and from $3x-1=-6$, we get $x=-\frac{5}{3}$. Our answers are $x=-\frac{5}{3}$ or $x=\frac{7}{3}$.

2. Solve $\dfrac{3-|y+5|}{2}=1$:

   We begin solving $\dfrac{3-|y+5|}{2}=1$ by isolating the absolute value to put it in the form $|X|=c$.

$$\begin{aligned}
\frac{3-|y+5|}{2} &= 1 \\
3-|y+5| &= 2 \quad \text{Multiply both sides by 2} \\
-|y+5| &= -1 \quad \text{Subtract 3 from both sides} \\
|y+5| &= 1 \quad \text{Divide both sides by } -1
\end{aligned}$$

   At this point, we have $y+5=1$ or $y+5=-1$, so our solutions are $y=-4$ or $y=-6$.

3. Solve $3|2t+1|-3=0$:

   As in the previous example, we first isolate the absolute value.

$$\begin{aligned}
3|2t+1|-3 &= 0 \\
3|2t+1| &= 3 \quad \text{Add 3 to both sides} \\
|2t+1| &= 1 \quad \text{Divide both sides by 3}
\end{aligned}$$

   From here, we have that $2t+1=1$ or $2t+1=-1$.

   Solving $2t+1=1$ gives $t=0$ and solving $2t+1=-1$ gives $t=-1$. Our answers are $t=-1$ or $t=0$.

EXAMPLE: Solve the following inequalities. Write your answer using interval notation.

1. Solve $|x-5|>1$:

   $|x-5|>1$ is equivalent to $x-5<-1$ or $x-5>1$.

   Solving this compound inequality, we get $x<4$ or $x>6$, which, in interval notation is: $(-\infty,4)\cup(6,\infty)$.

2. Solve: $\dfrac{4-2|2x+1|}{4}\geq -3$

$$\begin{array}{rcll} \dfrac{4-2|2x+1|}{4} & \geq & -3 & \\ 4-2|2x+1| & \geq & -12 & \text{Multiply both sides by 4} \\ -2|2x+1| & \geq & -16 & \text{Subtract 4 from both sides} \\ |2x+1| & \leq & \dfrac{-16}{-2} & \text{Divide by } -2\text{, reverse the inequality} \\ |2x+1| & \leq & 8 & \text{Reduce} \end{array}$$

   The inequality $|2x+1|\leq 8$ is equivalent to the compound inequality: $-8\leq 2x+1\leq 8$.

   Solving $-8\leq 2x+1\leq 8$, we get $-\dfrac{9}{2}\leq x\leq\dfrac{7}{2}$, or in interval notation, $\left[-\dfrac{9}{2},\dfrac{7}{2}\right]$.